Technical Notes

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An Approximation for Fully Stalled Cascades

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Introduction

STALL inception and poststall behavior of axial-flow compressors are important aspects of engine stability in modern aircraft. Attempts to model this behavior must incorporate blade-row characteristics, including the fully stalled condition.[1-3] These characteristics are often based on quasisteady, two-dimensional cascades, with an approximate dynamic response function.

However, information relating to fully stalled cascades is very limited, at best. Because of difficulties with separated flow and past interests, experiments usually have been confined to the normal operating range.[4] Analyses are being developed for separated flow,[5,6] but these are approximate, involved, and still not directed at the fully stalled condition. As a result of the limited information, very simple approximations, such as assumed constant values or isolated airfoil data, are sometimes used.

The purpose of this Note is to suggest an approximation for fully stalled cascade characteristics that approaches the correct limit as the through-flow is reduced to zero. The simple analysis is based on a control-volume approach with the flow assumed separated from the leading edge. Since the details of the flow are not calculated, reasonable approximations must be included for the mixing that occurs within the blade passages as well as in the wake. A number of possible assumptions have been considered, and the most realistic ones have been selected. The approach is similar to the well-known estimate for incidence losses in pumps and fans,[7] except that here the flow is not assumed to reattach to the blades.

Analysis

For convenience, the blade forces are considered parallel and normal to the chord line. The control volume for one blade passage extends from the undisturbed upstream flow to the trailing-edge plane (Fig. 1). The flow is assumed separated from the leading edge, as indicated by the dashed line in Fig. 1. At the trailing-edge plane there is a region of flow with velocity V_2 and a region with essentially zero velocity. Otherwise, the extent of the separated region does not affect the analysis. The momentum equation for this control volume gives

$$C_n = \frac{F_n}{\rho V_1^2 c/2} = \frac{2}{\sigma}\cos\beta_1\left(\sin\alpha - \frac{V_2}{V_1}\sin\delta\right) + \left(\frac{p_2 - p_1}{\rho V_1^2/2}\right)\frac{\sin\gamma}{\sigma} \tag{1}$$

and

$$C_t = \frac{F_t}{\rho V_1^2 c/2} = \frac{2}{\sigma}\cos\beta_1\left(\frac{V_2}{V_1}\cos\delta - \cos\alpha\right) + \left(\frac{p_2 - p_1}{\rho V_1^2/2}\right)\frac{\cos\gamma}{\sigma} \tag{2}$$

The flow angle δ, measured from the chord line, must be assumed. For closely-spaced blades one would not expect this angle to be large at the trailing edge. Near the pressure surface of the blades, the velocity should be parallel to the surface. In the shear layer between the separated and outer flows, the direction would be that of the displacement thickness on the suction surface. Thus, an angle of zero might be a reasonable approximation. The fully mixed angle at station 3, and actual deviation, is much larger than the trailing-edge value because of the mixing in the wake with constant momentum in the y direction.

Since the actual flow at the trailing-edge plane is not uniform, V_2 represents the momentum per unit mass flow. To account for the actual losses that occur within the blade passages due to the mixing, a loss factor is introduced.

$$\frac{p_2 - p_1}{\rho\, V_1^2/2} = 1 - (1+k)\frac{V_2^2}{V_1^2} \tag{3}$$

For relatively thin, slightly cambered blades, the sum of the pressure and shear force parallel to the chord line is assumed to be zero. This assumption implies that the force found by integrating the pressures along the chord line is very nearly equal to the total force and, again, is consistent with the present approximation. The velocity V_2 can then be

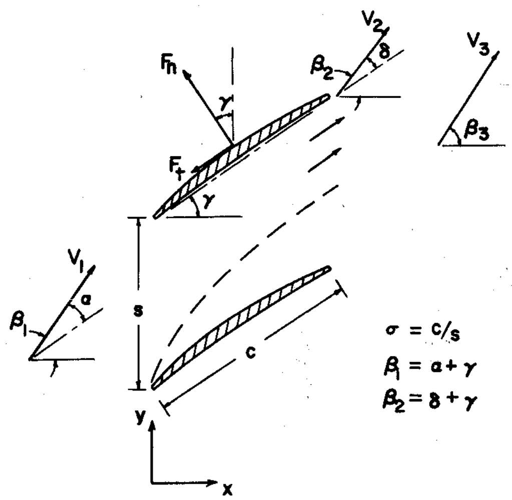

Fig. 1 Cascade geometry and nomenclature.

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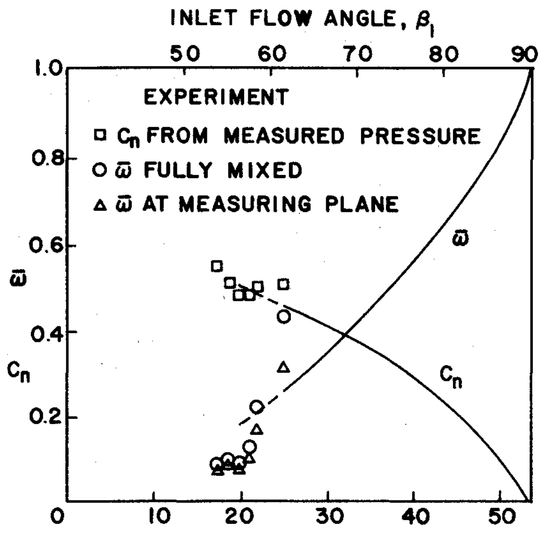

Fig. 2 Normal-force and loss coefficients.

determined from Eqs. (2) and (3),

$$\frac{V_2}{V_1}=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \tag{4}$$

where

$$a=1+k$$

$$b=\frac{-2\cos\beta_1\cos\delta}{\cos\gamma} \qquad c=\frac{2\cos\beta_1\cos\alpha}{\cos\gamma}-1$$

[To give meaningful results, only the positive value in Eq. (4) is used.]

For fully mixed, uniform flow at station 3 (with constant-area mixing), the flow angle can be determined from the mass flow rate and momentum in the y direction.

$$\tan\beta_3=\frac{V_2\sin\beta_2}{V_1\cos\beta_1} \tag{5}$$

The fully mixed pressure at station 3 can be determined from the mass flow rate and momentum in the x direction,

$$\frac{p_2-p_3}{\rho V_1^2/2}=2\cos\beta_1\left(\cos\beta_1-\frac{V_2}{V_1}\cos\beta_2\right) \tag{6}$$

Finally, the loss coefficient can be found from Eqs. (3) and (6),

$$\bar{\omega}=\frac{p_{01}-p_{03}}{\rho V_1^2/2}$$
$$=(1+k)\frac{V_2^2}{V_1^2}+2\cos\beta_1\left(\cos\beta_1-\frac{V_2}{V_1}\cos\beta_2\right)-\frac{\cos^2\beta_1}{\cos^2\beta_3} \tag{7}$$

To close the system of equations, an approximation must be made for the relative total pressure loss within the blade passage, k. Since most of the losses occur in the free shear layer, the ℓ factor k is expected to be nearly proportional to its width and, therefore, the chord length, c. The relative losses should also vary inversely with the mass flow rate per blade passage, or $s\cos\beta_1$, since they represent the mass-averaged values. A tentative suggestion, with the constant based on very limited data, is given by

$$k=0.15\sigma/\cos\beta_1 \tag{8}$$

Results and Discussion

The above equations [(8), (4), (3), (1), (5), and (7)], which give an explicit relation for the loss and force coefficients, were solved in that order on a hand-held computer for the case considered in Ref. 6 (Fig. 2). The limited data, which involve some uncertainty because of three-dimensional and unsteady flow, simply indicate approximate limits the analysis should approach at low angles of attack. Unfortunately, most of the data is below the applicable range of the analysis (indicated by the dashed part of the curve), which is one reason for this Note. As might be expected, the trend in the data that involves at least some partially stalled flow does not follow that of the analysis. In particular, the analysis assumes that the flow is separated from the leading edge and is not very sensitive to the angle of attack.

Other possible assumptions, including constant pressure to the trailing edge and different approximations for the losses within the blade passages, were considered in the development. Based on the limited information and a very approximate consideration of free-shear-layer mixing, those selected gave the best results. With more complete data, however, Eq. (8) could be improved.

Although the method is very approximate, it does give reasonable results and approaches the correct limits at very high inlet flow angles. From a practical standpoint, it is probably within the accuracy that one can make use of a quasisteady, two-dimensional result.

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